Lect7

N-element Array

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N-element Array : Uniform Amplitude and spacing

Uniform Array:

- Identical elements with identical amplitudes
- Progressive phase shift

.

Why antenna Array

1-Usually gain of single element is low, thus array is used for **increasing gain** for long distance communication

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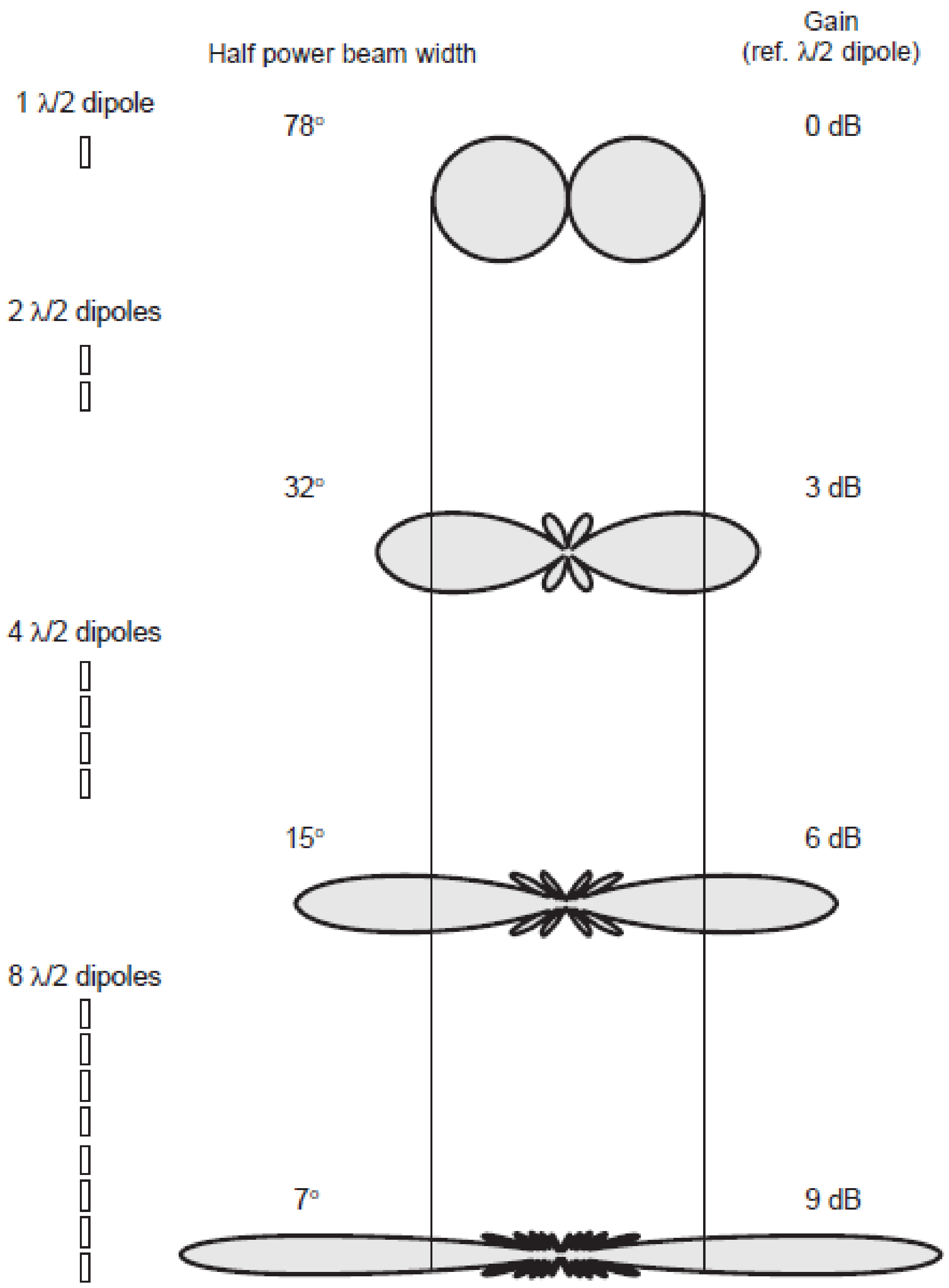
If λ/2 dipole is reference
i.e. its Gain considered to be=0dB
("note that λ/2 dipole has D=2dB")
then

2 element array increase gain by 3dB(double gain 2 time)

4 element array increase gain by 6dB(double gain 4 time)

8 element array could increase gain by 9dB(double gain 8time)

2-Beam steering
by changing progressive phase
3-Nulling interference directions



Broad side array β=0

- ***N-ELEMENT LINEAR ARRAY: Uniform Amplitude* and Spacing**

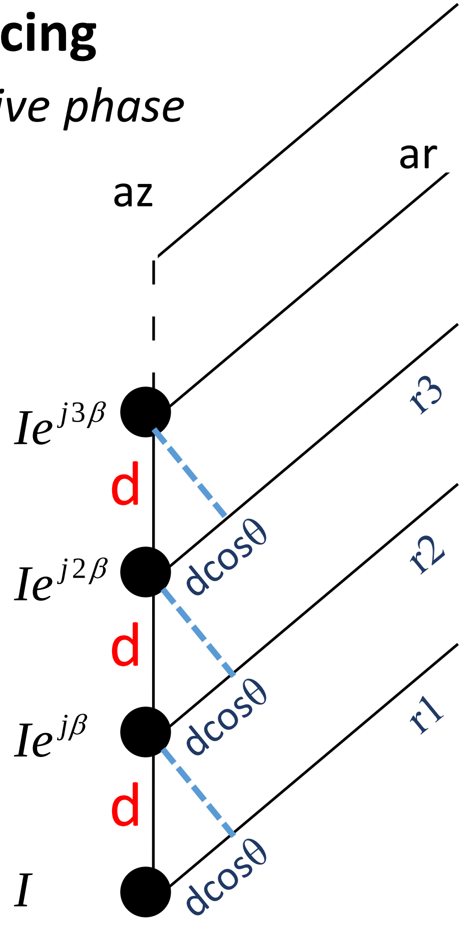
uniform array has: Identical elements-Identical magnitude-Progressive phase

Also uniform spacing

- $Etot=E1+E2+E3+....+EN$

- $Etot=A\,[I_1 e^{-jkr1}+ I_2 e^{-jkr2} + I_3 e^{-jkr3} + ...+ I_n e^{-jkrN}]$
- Where $A=(j\eta kL/4\pi r)\sin\theta$ for infinitesimal dipole
- $I_2= I_1 e^{j\beta}$
- $r_2 = r_1 - d\cos\theta$

- $Etot=A\, I_1 e^{-jkr1}[1+ e^{j\beta} e^{jkd\cos\theta} + e^{j2\beta} e^{j2kd\cos\theta} + e^{j3\beta} e^{j3kd\cos\theta}+....]$



$$AF=1+ e^{j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + e^{j3(kd\cos\theta+\beta)} +.....+ e^{j(N-1)(kd\cos\theta+\beta)}$$

$$AF=1+ e^{j\psi} + e^{j2\psi} + e^{j3\psi} +.....+ e^{j(N-1)\psi}$$

Where, $\psi=kd\cos\theta+\beta$

- **AF = 1+ $e^{j\psi}$ +$e^{j2\psi}$ +$e^{j3\psi}$ +…..+$e^{j(N-1)\psi}$** *(1)*
- **AF. $e^{j\psi}$ = $e^{j\psi}$ +$e^{j2\psi}$ +$e^{j3\psi}$ +…..+$e^{j(N-1)\psi}$+ $e^{jN\psi}$** *(2)*
- **Subtract (1) from (2)** **AF$(e^{j\psi} - 1) = (-1+ e^{jN\psi})$**

$$\text{AF} = \left[\frac{e^{jN\psi} - 1}{e^{j\psi} - 1}\right] = e^{j[(N-1)/2]\psi}\left[\frac{e^{j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{j(1/2)\psi} - e^{-j(1/2)\psi}}\right]$$

$$= e^{j[(N-1)/2]\psi}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

Max occurred at $\text{AF} = \frac{0}{0}$ **which occurred at $\psi/2 = \pm m\pi$ (for m=0,1,2,.)**

To get Max value differentiate num and denum w.r.t. ψ *(and substitute ψ=0)*

AF_{MAX}= N hence

$$(\text{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

Observations:

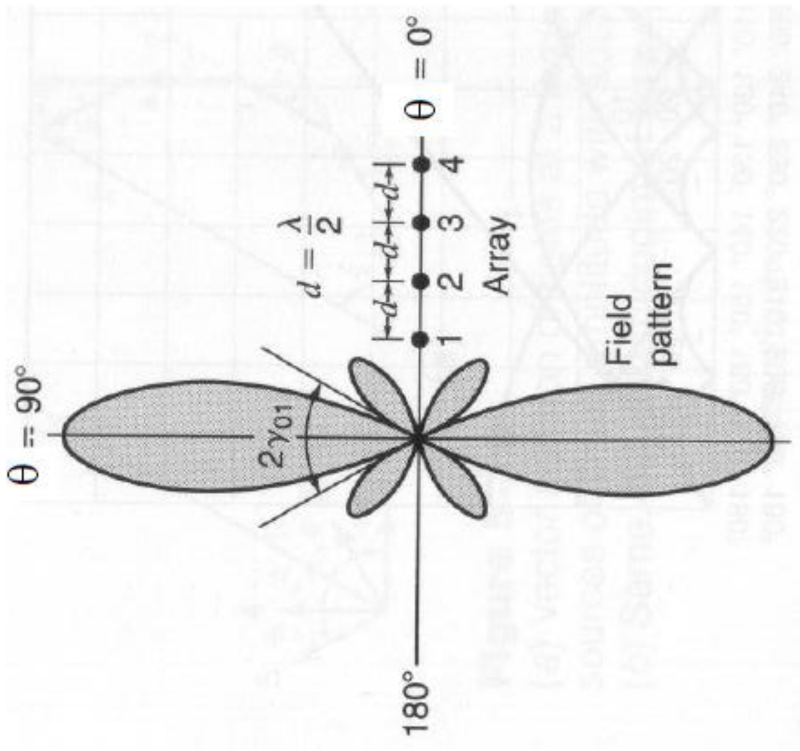
(1) Main lobe is in the direction so that

$\psi = k\, d \cos\theta + \beta = 0$

(2) The main lobe narrows as N increases.

- Max occurred at $\psi = 0 = k.d.\cos\theta + \beta$ (for AF pattern)

Broad Side Array

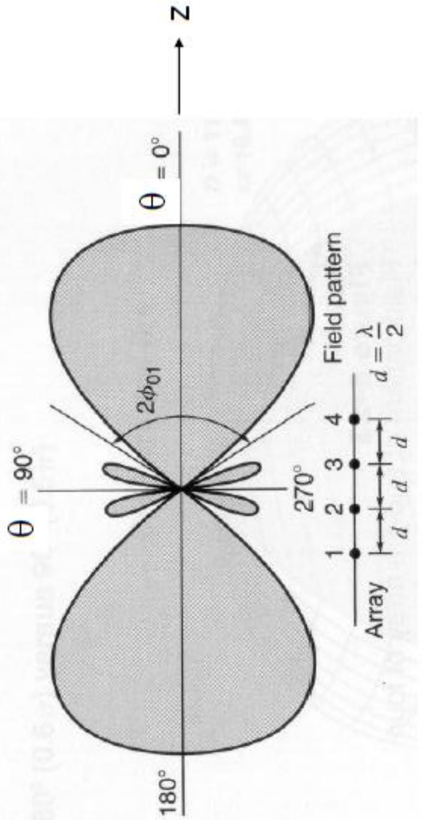


Since it is desired to have the first maximum directed toward $\theta_0 = 90^\circ$, then

Setting For broad side AF pattern

$$\psi = kd\cos\theta + \beta|_{\theta=90^\circ} = \beta = 0$$

End Fire Array



Setting For End Fire AF pattern

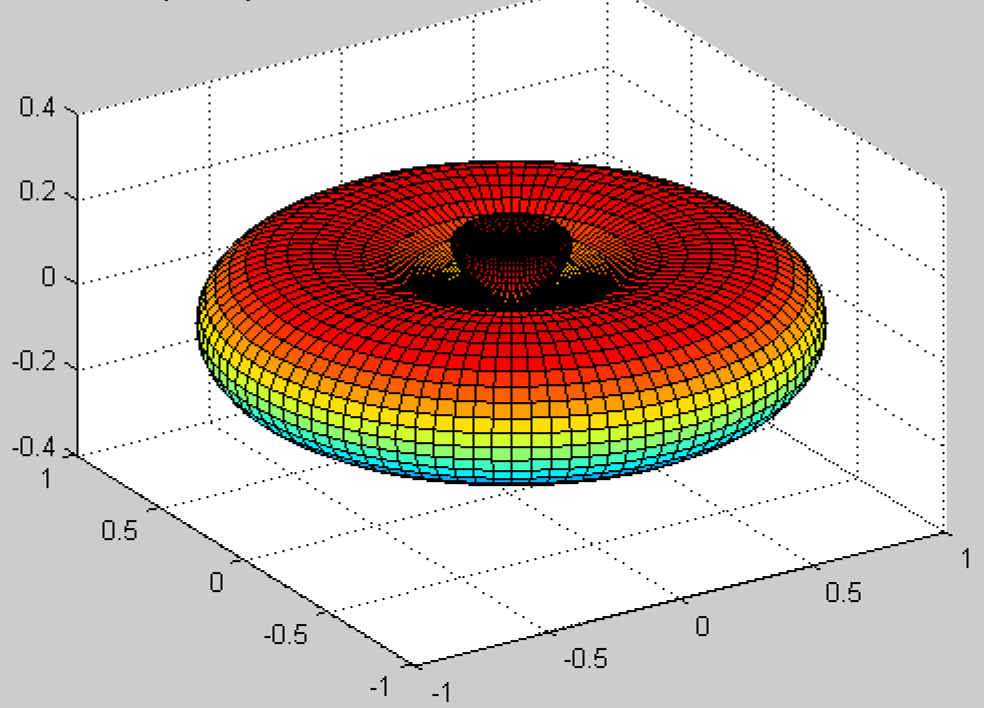
To direct the first maximum toward $\theta_0 = 0^\circ$,

$$\psi = kd\cos\theta + \beta|_{\theta=0^\circ} = kd + \beta = 0 \Rightarrow \beta = -kd$$

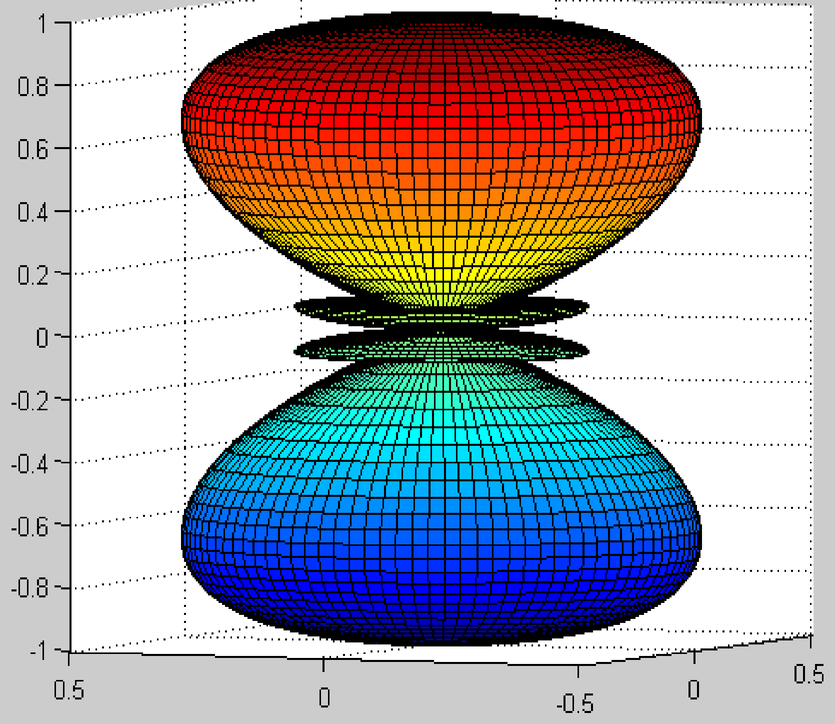
If the first maximum is desired toward $\theta_0 = 180^\circ$, then

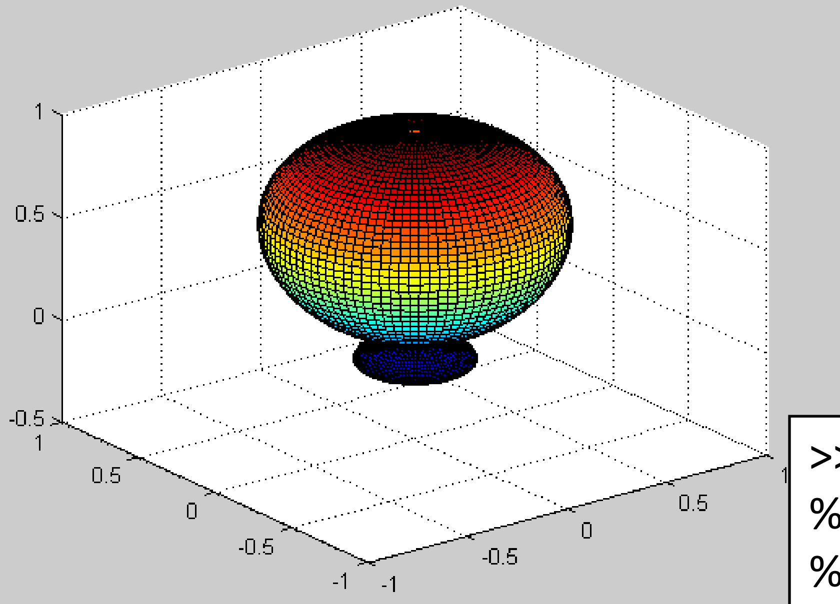
$$\psi = kd\cos\theta + \beta|_{\theta=180^\circ} = -kd + \beta = 0 \Rightarrow \beta = kd$$

broadside(4,.5)



>> endfire(4,.5)





>> endfire(4,.25)
%endfire(N,d)
%note for d<λ/4 only one main lobe directed at θ=0

It is required to study $(AF)_n$

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- **Nulls**

$$N\frac{\psi}{2} = \pm m\pi \quad , \quad m = 1,2,3,.. \neq 0, N, 2N, ...$$

- **Maximum**

$$\frac{\psi}{2} = \pm m\pi \quad , \quad m = 0,1,2,....(\,0 \text{ for main lobe }\,)$$

Grating lobe condition (at m=1,2,3,...)

- **3-dB point**

$$N\frac{\psi}{2} = \pm 1.39$$

- **Secondary Maximum for minor lobes**

$$N\frac{\psi}{2} = \pm\frac{2s+1}{2}\pi \qquad , \quad s = 1, 2, 3, ...$$

Maximum of first minor lobe occurred at $N\psi/2=\pm 3\pi/2$

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- NULLS
- Nulls occurred at *sin(Nψ/2)*=0
- Kdcosθ+ β=±2nπ/N where n=1,2,3 (again n≠ 0 or N or 2N…….this make $(AF)_n$=0/0 which is max condition)

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

- **Broadside Array (sources in phase β=0)**

$$\theta_n = \cos^{-1}\left(\pm\frac{n}{N}\frac{\lambda}{d}\right)$$
$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

End fire Array (β=-kd)

$$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$$
$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

1- because $\cos^{-1}$(less than 1)

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- MAXIMUM
- Maximum occurred at $\psi/2 = \pm m\pi$ (for $(AF)_n$=0/0)
- Kdcosθ+ β=±2mπ where m=0,1,2,3

$$\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$

- For sin(Nx)/Nsin(x)maximum occurred at x=0 or $\psi/2=0$ *i.e* m=0
-
- **Broa**

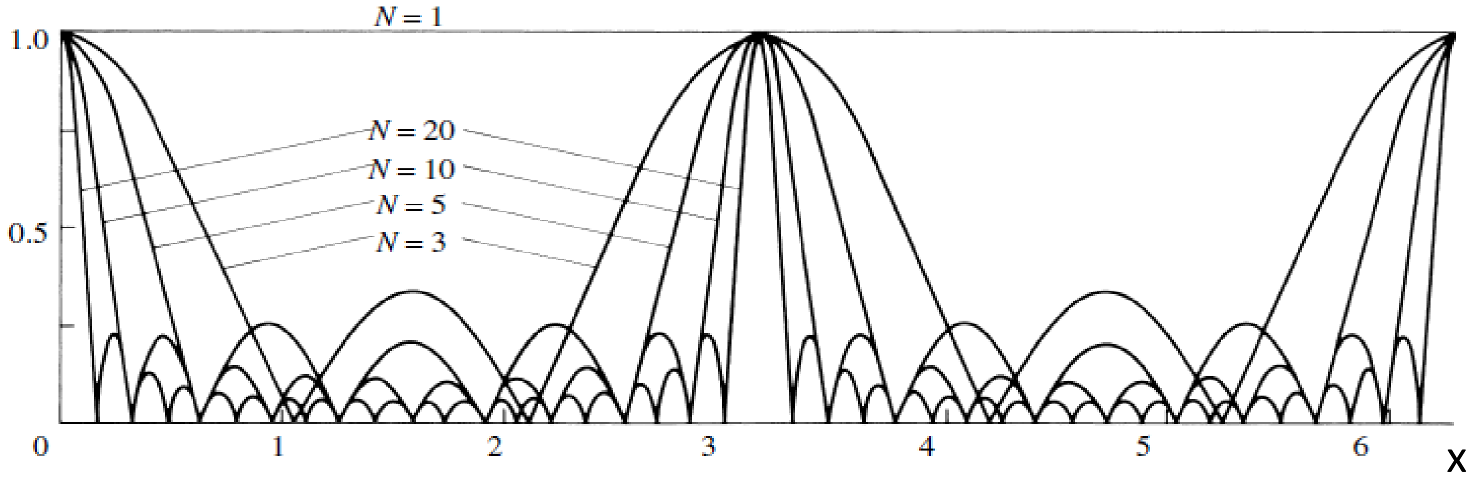


Figure II.1 Curves of $|\sin(Nx)/N\sin(x)|$ function.

Grating lobe condition

- Grating Lobe is lobe with Maxima(as of major) in other direction (un required direction).

Max condition array factor was(0/0 condition) at $\psi/2=0$ ***i.e*** $Kd\cos\theta_g+\beta=\pm 2m\pi$,

- **for broad side** $\beta=0$

$\theta_m=\cos^{-1}(m\lambda/d)$ there is no grating lobe as long as $d_{max}<\lambda$

$\cos^{-1}(m\lambda/d)$ exist only at $m=0$ when $d_{max}<\lambda$

if d realize $m\lambda/d<1$ or $d>m\lambda$ there is grating lobe.

- **For end fire**, $\beta=-kd$

for $\theta_m=\cos^{-1}(1-n\lambda/d)$ there is no grating lobe as long as $d_{max}<\lambda/2$

- **$\cos^{-1}(1-n\lambda/d)$ exist only at $m=0$ when $d_{max}<\lambda/2$**

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- <u>3-dB point for AF</u>
- <u>Use Approximation</u> sin(x)/x because it does not depend on N

Using try and error 3dB occurred at sin(x)/x=.707 i.e. x=1.39

*because it is field pattern (sin(1.93*180/π)/1.39=.7076*

x	$\sin(x)/x$
1.3	0.74120
1.4	0.70389

$$\frac{N}{2}\psi = \frac{N}{2}(kd\cos\theta + \beta)|_{\theta=\theta_h} = \pm 1.391$$

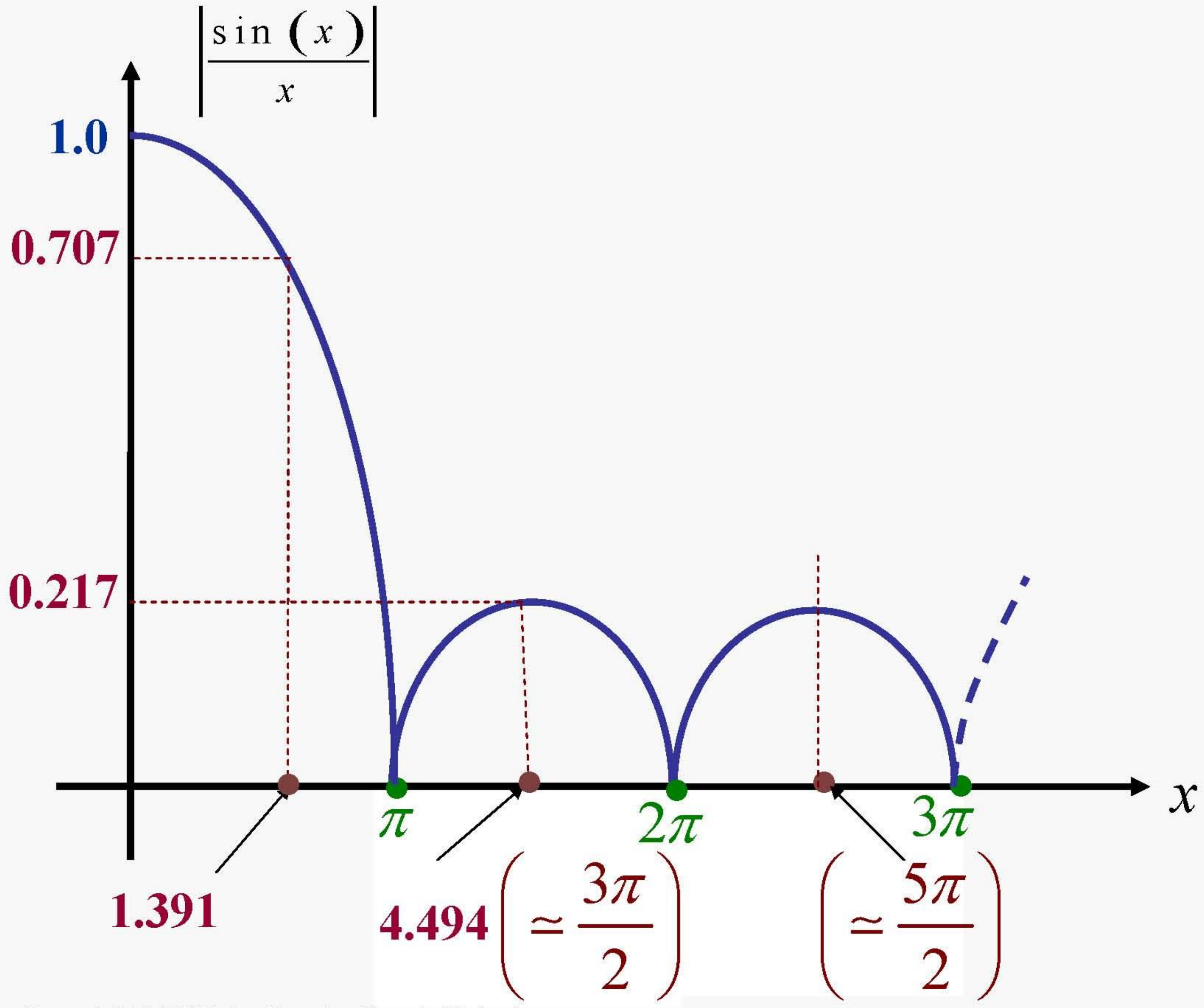
$$\Rightarrow \theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

- **Broadside Array (sources in phase β=0)**

HALF-POWER POINTS $\quad \theta_h \simeq \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi N d}\right)$

End fire Array (β=-kd)

$$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$$


$\left|\frac{\sin(x)}{x}\right|$
1.0
0.707
0.217
x
π
2π
3π
1.391
$4.494 \left(\simeq \frac{3\pi}{2}\right)$
$\left(\simeq \frac{5\pi}{2}\right)$

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- **Secondary Maximafor AF**

Secondary maxima occurred at numerator is maxima

$$\sin\left(\frac{N}{2}\psi\right) = \sin\left[\frac{N}{2}(kd\cos\theta+\beta)\right]\bigg|_{\theta=\theta_s} \simeq \pm 1 \Rightarrow \frac{N}{2}(kd\cos\theta+\beta)|_{\theta=\theta_s}$$
$$\simeq \pm\left(\frac{2s+1}{2}\right)\pi \Rightarrow \theta_s \simeq \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[-\beta \pm \left(\frac{2s+1}{N}\right)\pi\right]\right\}$$
$$s = 1, 2, 3, \ldots$$

- Maximum of first minor lobe occurred at Nψ/2=3π/2 (i.e. s=1)

$$(AF)_n = \frac{\sin(\frac{N\psi}{2})}{\frac{N\psi}{2}} = \frac{1}{3\pi/2} = .212 = -13dB$$

20log() as it is amplitude

- **Broadside Array (sources in phase β=0)**

MINOR LOBE MAXIMA

$$\theta_s \simeq \cos^{-1}\left[\pm\frac{\lambda}{2d}\left(\frac{2s+1}{N}\right)\right]$$
$$s = 1, 2, 3, \ldots$$

End fire Array (β=-kd)

$$\theta_s \simeq \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right]$$
$$s = 1, 2, 3, \ldots$$

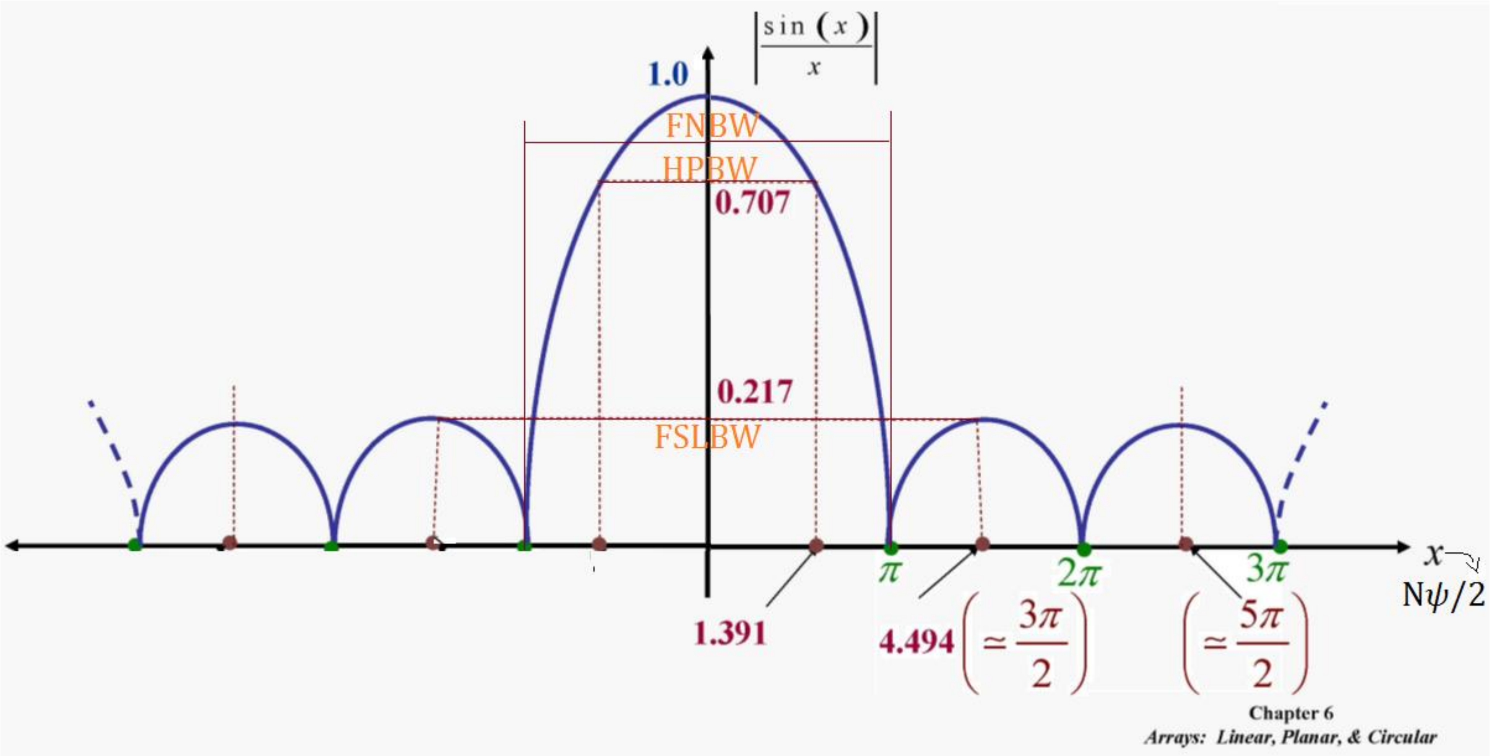


TABLE 6.2 Beamwidths for Uniform Amplitude Broadside Arrays

FIRST-NULL BEAMWIDTH (FNBW)	$\Theta_n = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right]$
HALF-POWER BEAMWIDTH (HPBW)	$\Theta_h \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi Nd}\right)\right]$ $\pi d/\lambda \ll 1$
FIRST SIDE LOBE BEAMWIDTH (FSLBW)	$\Theta_s \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3\lambda}{2dN}\right)\right]$ $\pi d/\lambda \ll 1$

TABLE 6.4 Beamwidths for Uniform Amplitude Ordinary End-Fire Arrays

FIRST-NULL BEAMWIDTH (FNBW)	$\Theta_n = 2\cos^{-1}\left(1 - \frac{\lambda}{Nd}\right)$
HALF-POWER BEAMWIDTH (HPBW)	$\Theta_h \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi dN}\right)$ $\pi d/\lambda \ll 1$
FIRST SIDE LOBE BEAMWIDTH (FSLBW)	$\Theta_s \simeq 2\cos^{-1}\left(1 - \frac{3\lambda}{2Nd}\right)$ $\pi d/\lambda \ll 1$

adjusted so that

$$\psi = kd\cos\theta + \beta|_{\theta=\theta_0} = kd\cos\theta_0 + \beta = 0 \Rightarrow \beta = -kd\cos\theta_0$$

For Broadside array β=0, d=λ

$\theta_m = \cos^{-1}(\pm m\,\lambda/d) = \cos^{-1}(\pm m)$

exist at

m=0(main lobe) $\theta_m = \pi/2$

m=1 $\theta_m = 0, \pi$

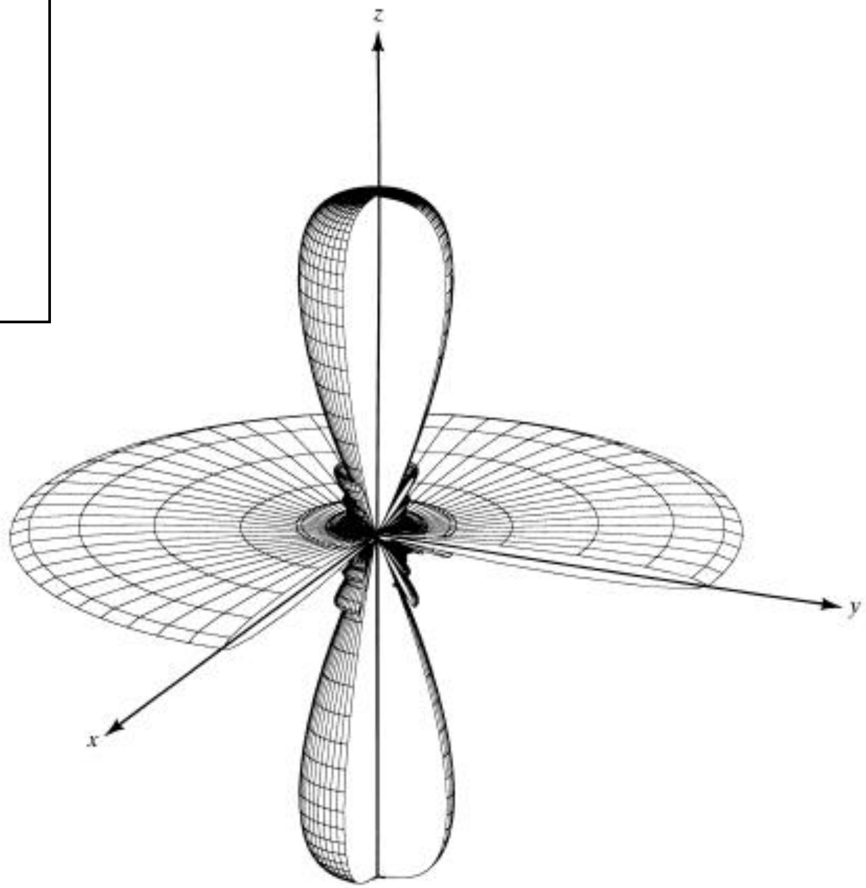
HPBW=5.07 ° for main lobe

HPBW=34.4 ° for grating lobe

Try and error for θh for grating lobe

Using

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$



(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

Figure 6.6 Three-dimensional amplitude patterns for broadside, and broadside/end-fire arrays ($N = 10$).

Broadside Array

For d=λ maxima at 0,90,180

d=λ/4 maxima at 90

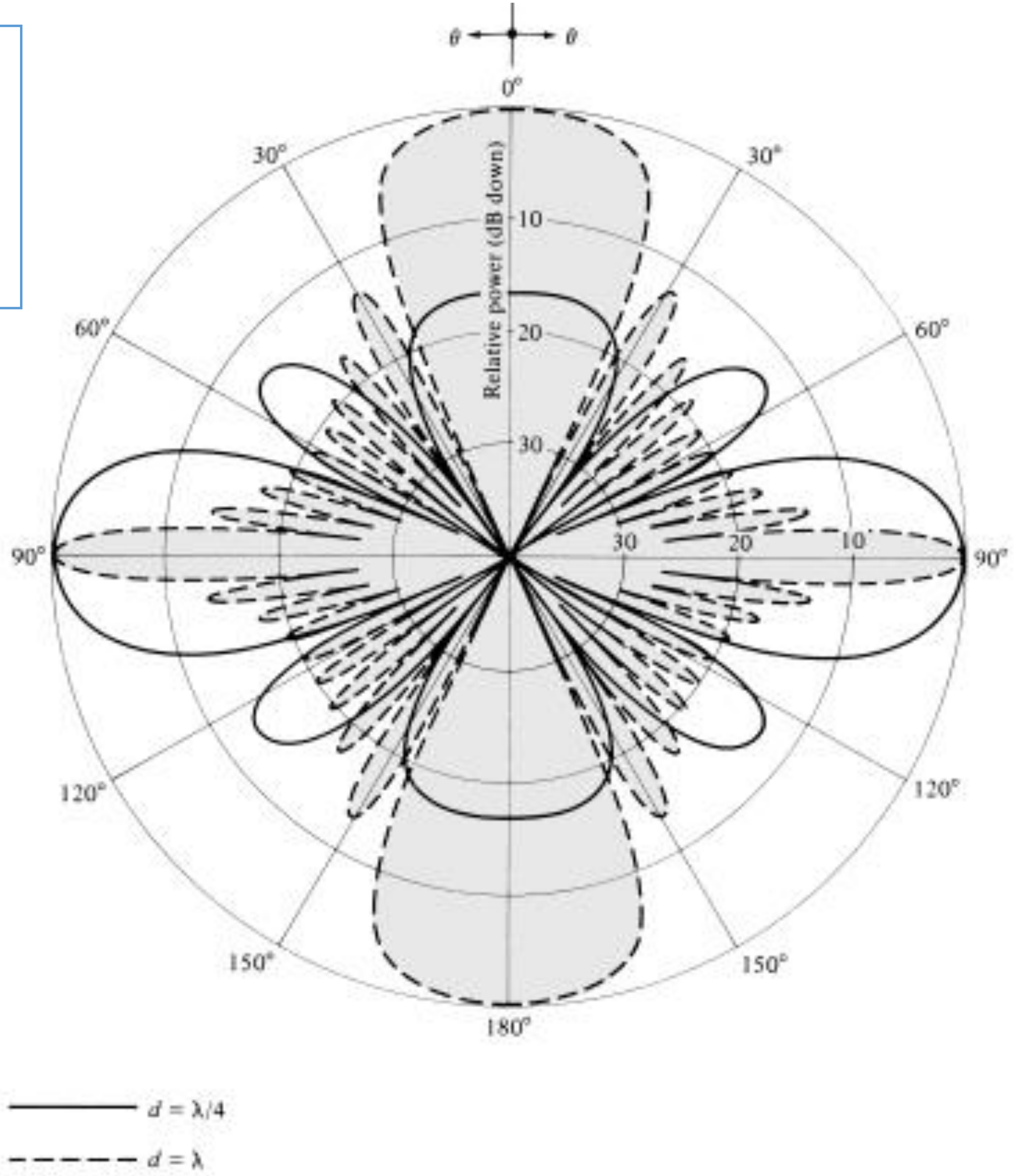


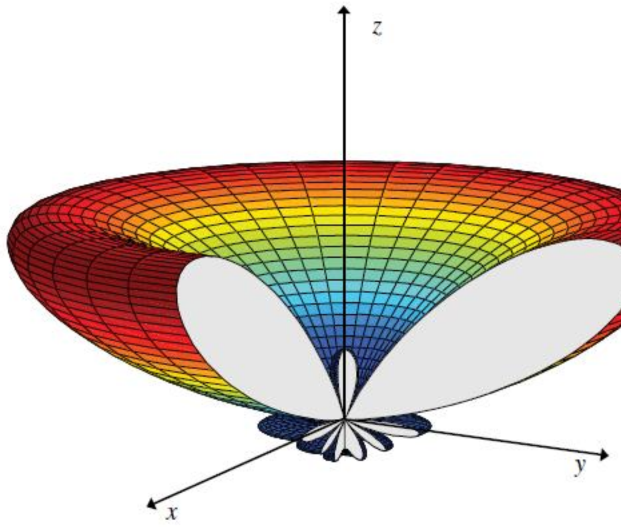
Figure 6.7 Array factor patterns of a 10-element uniform amplitude broadside array ($N = 10$, $\beta = 0$).

Directivity:

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi}(AF)_n^2 \sin\theta \, d\theta \, d\emptyset}$$

Note that Krause approximation is not valid here as radiation may not have single lobe with two perpendicular planes That contain max radiation. Figure below illustrate this concept:



Example : Problem 6.17

Design a 19 element uniform linear scanning array with a spacing of $\lambda/4$ between elements

(a) What is the progressive phase excitation between elements so that maximum of the array factor is 30^o From the line where the elements are placed

(b) What is the half power beam width of the array factor of part a

(c) What is the value (in dB) of the maximum of the first minor lobe

Solution:

Include computing directivity:-

6-17

$d = \lambda/4 \quad N = 19 \quad \theta_{max} = 30° \quad \beta = ??$

(a) max at $\frac{\psi}{2} = 0$

or $kd \cos\theta_{max} + \beta = 0 \qquad kd = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

$\beta = -\frac{\pi}{2} \cos 30 = -\frac{\sqrt{3}\,\pi}{4} = -1.36$ rad

(b) θ_h at $\frac{N\psi}{2} = \pm 1.391$

$kd \cos\theta_h - 1.36 = \pm \frac{2}{19} \times 1.391$

$\theta_h = \cos^{-1} \frac{2}{\pi}\left[\pm \frac{2}{19} \times 1.391 + 1.36 \right] \rightarrow \begin{cases} \theta_{h_1} = 0.2873 \text{ rad} \\ \theta_{h_2} = 0.68788 \end{cases}$

$\therefore \text{HPBW} = |\theta_{h_2} - \theta_{h_1}| = 0.4 \text{ rad} = 23°$

if we want to Compute D

if we want to compute D

$$D = \frac{2}{\int_0^{\pi} \left[\frac{\sin\left(\frac{19}{2}\left[\frac{\pi}{2}\cos\theta - 1.36\right]\right)}{19 \sin\left(0.5\left(\frac{\pi}{2}\cos\theta - 1.36\right)\right)} \right]^2 \sin\theta \, d\theta} = \frac{2}{0.195}$$

$D = 10.25 = 10.1$ dB .

(c) First minor lobe occure at $\frac{N\psi}{2} = \frac{3\pi}{2}$

$$\therefore (AF)_n = \frac{\sin\frac{3\pi}{2}}{19 \sin\left(\frac{3\pi}{2 \times 19}\right)} = -0.214 \xrightarrow[20\log]{|\,|} \therefore \text{in dB } (AF)_n = -13.37 \text{ d}$$

#

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

Axis

$z:\quad \psi = kd\cos\theta + \beta$

$x:\quad \psi = kd\sin\theta\cos\phi + \beta$

$y:\quad \psi = kd\sin\theta\sin\phi + \beta$

example **_Draw Radiation pattern of AF_**

$$array\ on\ y\quad axis,\ N=6,\ \theta_{\max}=30^{o},-\pi<\theta<\pi, dx=dy=\lambda/2$$

$$\beta_x=\beta_y=-kd\sin\theta_{\max}=-0.5\pi$$

Soln:

Get nulls:

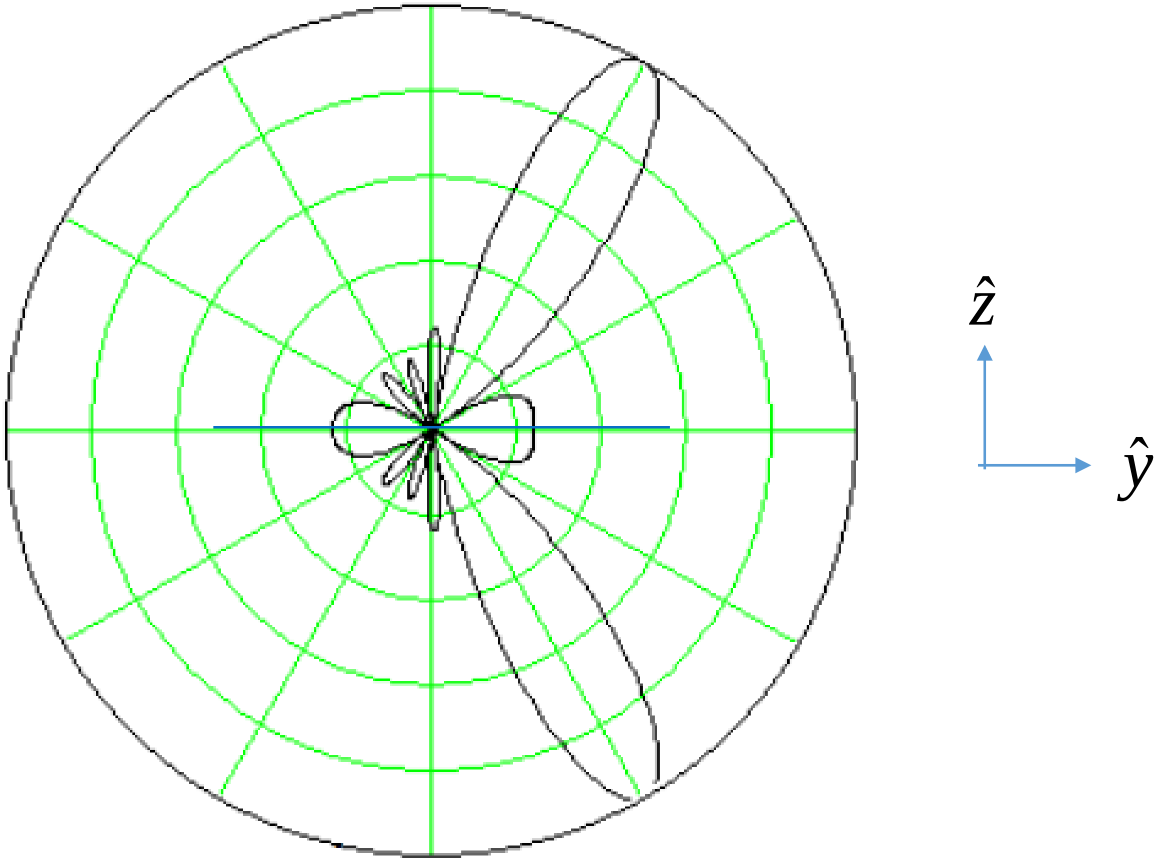
$$AF=\frac{\sin(3\pi(\sin\theta-0.5))}{6\sin(0.5\pi(\sin\theta-0.5))}$$

$$\theta_{null}=(\pm\frac{m}{3}+.5)\quad m=1\rightarrow\theta_{null}=56.4^{\circ},9.6^{\circ}$$

$$m=2\rightarrow\theta_{null}=-9.6^{\circ}$$

$$.m=3\rightarrow\theta_{null}=-30^{\circ}$$

$$.\qquad m=4\rightarrow\theta_{null}=-56.4^{\circ}$$



6.10 PLANAR ARRAY

Planar arrays can provide more symmetrical patterns with lower side lobes.

Array Factor AF=AFx.AFy

M elements along x-axis :The spacing and progressive phase shift between the elements along the *x*-axis are represented, respectively, by *dx* and βx.

$$AF_x = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)}$$

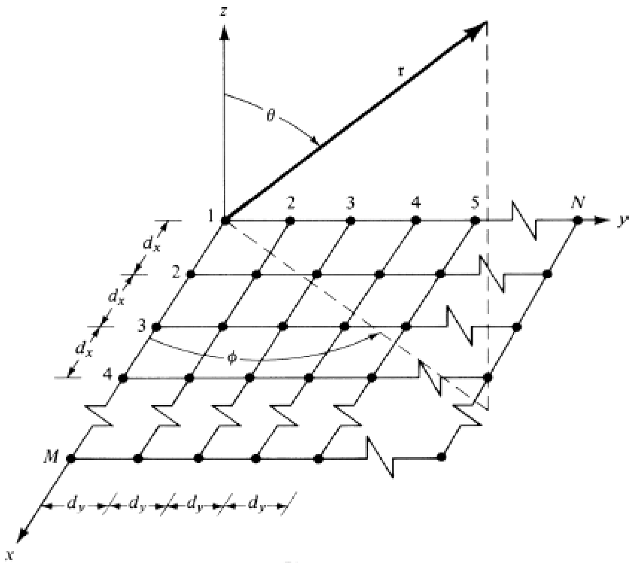
and N elements along y axis a distance *dy* apart and with a progressive phase βy.

$$AF_y = \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)}$$

$$AF_n(\theta,\phi) = \left\{\frac{1}{M}\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\}$$

where

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x$$
$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y$$



(b) Planar array

Figure 6.30 Linear and planar array geometries.

To avoid grating lobes in the *x-z* and *y-z* planes, the spacing between the elements in the *x-* and *y-*directions, respectively, must be less than $\lambda/2$ ($dx < \lambda/2$ and $dy < \lambda/2$).

For a rectangular array, the major lobe and grating are located at

$kd_x \sin\theta \cos\phi + \beta_x = \pm 2m\pi \qquad m = 0, 1, 2.....$

$kd_y \sin\theta \sin\phi + \beta_y = \pm 2n\pi \qquad n = 0, 1, 2,...$

The phases βx and βy are independent of each other, and they can be adjusted so that the main beam of *AFx* is not the same as that of *AFy*. However, in most practical applications it is required that the conical main beams of *AFx* and *AFy* intersect and their maxima be directed toward the same direction.

For one main beam that is directed along $\theta = \theta_0$ and $\phi = \phi_0$, the

$$\beta_x = -kd_x \sin\theta_0 \cos\phi_0$$

$$\beta_y = -kd_y \sin\theta_0 \sin\phi_0$$

Solving simultaneously

$$\tan\phi_0 = \frac{\beta_y d_x}{\beta_x d_y}$$

$$\sin^2\theta_0 = \left(\frac{\beta_x}{kd_x}\right)^2 + \left(\frac{\beta_y}{kd_y}\right)^2$$

PLANAR ARRAY Directivity

The directivity of the array factor AF(θ, ϕ) whose major beam is pointing in the $\theta = \theta 0$ and $\phi = \phi 0$ direction, can be obtained by:

$$D_0 = \frac{4\pi}{\displaystyle\int_0^{2\pi}\int_0^{\pi} \frac{[\mathrm{AF}(\theta,\phi)][\mathrm{AF}(\theta,\phi)]^* \sin\theta}{[\mathrm{AF}(\theta_0,\phi_0)][\mathrm{AF}(\theta_0,\phi_0)]^*|_{\max}}\, d\theta\, d\phi}$$

$$\mathrm{AF}_n(\theta,\phi) = \left\{\frac{1}{M}\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\}$$

where

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x$$

$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y$$

For large planar arrays, which are nearly broadside, the directivity reduces to

$$D_0 = \pi\cos\theta_0 D_x D_y$$

$D_x = 2M(d_x/\lambda)$ and $D_y = 2N(d_y/\lambda)$

where *Dx* and *Dy* are the directivities of broadside linear arrays each, respectively, of number of elements *M* and *N*.

6.50. Design a 10 × 8 (10 in the x direction and 8 in the y) element uniform planar array so that the main maximum is oriented along $\theta_0 = 10°$, $\phi_0 = 90°$. For a spacing of $d_x = d_y = \lambda/8$ between the elements, find the

(a) progressive phase shift between the elements in the x and y directions

(b) directivity of the array

6-50. $d_x = d_y = \lambda/8, \quad M=10, \; N=8, \; \theta_0 = 10°, \; \phi_0 = 90°$

a. $\beta_x = -kd_x \sin\theta_0 \cos\phi_0 = -\frac{2\pi}{\lambda}\frac{\lambda}{8}\sin(10°)\cos(90°) = 0$

$\beta_y = -kd_y \sin\theta_0 \sin\phi_0 = -\frac{2\pi}{\lambda}\frac{\lambda}{8}\sin(10°)\sin(90°) = -0.1364 \text{ rad} = -7.81°$

b. $D_0 = \pi \cos\theta_0 \, D_x D_y$

$D_x = 2N\left(\frac{d_x}{\lambda}\right) = 2(10)\left(\frac{1}{8}\right) = 2.5 = 3.98 \text{ dB}$

$D_y = 2N\left(\frac{d_y}{\lambda}\right) = 2(8)\left(\frac{1}{8}\right) = 2.0 = 3.01 \text{ dB}$

$D = \pi\cos(10°)(2.5)(2) = 15.47 = 11.89 \text{ dB}$